EJERCICIO 1 (1,25)

$$\vec{r} = (2-2t^2)\,i + (5-5t^2)\,j \text{ (m)}$$

a) $\vec{v} = \dfrac{d(\vec{r})}{dt} = \dfrac{d((2-2t^2)i + (5-5t^2)j)}{dt}$ m/s

$= \boxed{(-4t\,i - 10t\,j) \text{ m/s}}$

Módulo $|\vec{v}| = \sqrt{(-4t)^2 + (-10t)^2}\ \text{m/s} = \sqrt{16t^2 + 100t^2}\ \text{m/s} = \sqrt{t^2(100+16)}$ m/s

$= \sqrt{116t^2}\ \text{m/s} = t\sqrt{116}\ \text{m/s} = \boxed{10{,}78t \text{ m/s}}$

b) $\vec{a} = \dfrac{d(\vec{v})}{dt} = \dfrac{d(-4t\,i - 10t\,j)}{dt}\text{ m/s} = (-4i - 10j)\text{ m/s}^2$

c) $\vec{a}$ no depende del tiempo es constante luego en $t=1$ y $t=3$ $\vec{a} = (-4i - 10j)\text{ m/s}^2$

d) $|\vec{a}| = \sqrt{(-4)^2 + (-10)^2}\ \text{m/s}^2 = 10{,}78 \text{ m/s}^2$

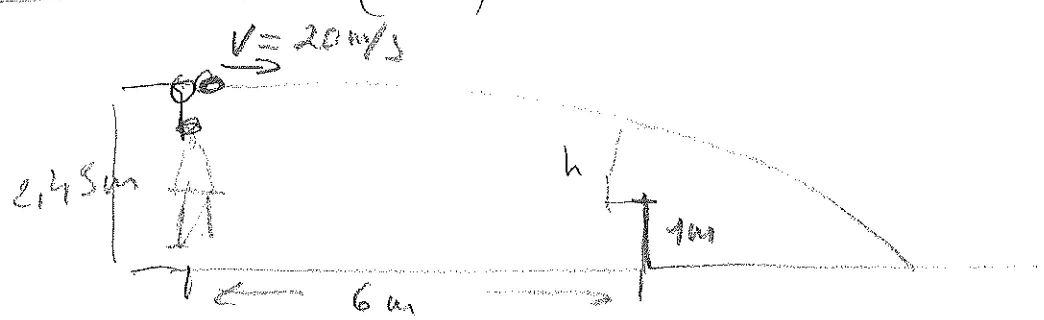
$|\vec{a}_t| = \dfrac{d|\vec{v}|}{dt} = \dfrac{d(10{,}78t)}{dt}\text{ m/s}^2 = 10{,}78 \text{ m/s}^2$

$|\vec{a}|^2 = |a_t|^2 + |a_n|^2 \Rightarrow |a_n|^2 = |\vec{a}|^2 - |\vec{a}_t|^2 =$

$(10{,}78 \text{ m/s}^2)^2 - (10{,}78 \text{ m/s}^2)^2 = 0 \text{ m/s}^2$

$|\vec{a}_n| = \boxed{0 \text{ m/s}^2}$. No hay aceleración normal.

EJERCICIO 2 (1,5)



a) $x = V_x \cdot t \Rightarrow$ Ecuación de movimiento en el eje x

$$6m = 20 m/s \cdot t \Rightarrow t = \frac{6m}{20 m/s} = 0,3 s$$

La posición en el eje Y se obtiene mediante la expresión

$$Y = Y_0 + \cancel{V_{0y} t} - \frac{1}{2} g t^2$$

$$= 2,45 m - \frac{1}{2} 9,8 m/s^2 \cdot 0,3^2 = 2m$$

la pelota pasa a 2m − 1m = $\boxed{\text{1m de la red}}$

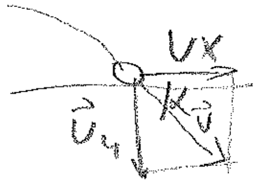
b) Tiempo que tarda en llegar al suelo se obtiene de la expresión de caída libre

$$y = y_0 - \frac{1}{2} g t^2$$

$$0 m = 2,45 m - \frac{1}{2} 9,8 m/s^2 t^2 = 2,45 m - 4,9 m/s^2 t^2$$

$$\Rightarrow t = \sqrt{\frac{2,45}{4,9}} s = 0,7 s$$

c)



$$V_x = 20 m/s$$

$$V_y = -gt = -9,8 m/s^2 \cdot 0,7 s = -6,86 m/s$$

$$\vec{V} = 20 \vec{i} - 6,86 \vec{j} \ (m/s)$$

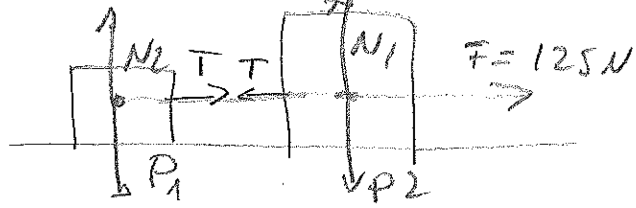
$$\alpha = \text{arctg} \frac{-6,86}{20} = -18,9^\circ$$

$$|\vec{V}| = \sqrt{20^2 + (-6,86)^2} = \boxed{21,14 m/s}$$

$$\vec{V} = 21,14_{-18,9^\circ} m/s$$

EJERCICIO 3 (1,25)

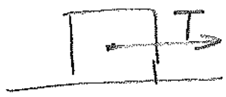
c) $m_1 = 15\,kg$ $m_2 = 25\,kg$



a)

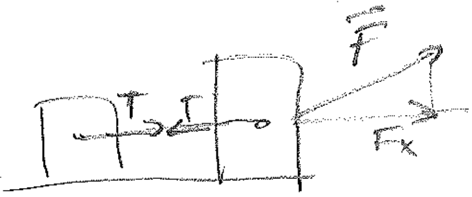
$$F = m \cdot a$$
$$125\,N = (15\,kg + 25\,kg) \cdot a$$
$$a = \frac{125}{40}\,m/s^2 = 3{,}125\,m/s^2$$

b) $m_1 = 15\,kg$



$$T = m_1 \cdot a = 15\,kg \cdot 3{,}125\,m/s^2 = \boxed{46{,}9\,N}$$

d)

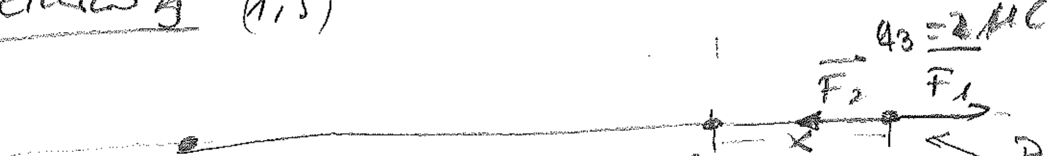


$$F_x = F \cdot \cos 25° = 125 \cdot 0{,}91\,N = 113\,N$$

$$F_x = m \cdot a$$
$$113\,N = 40\,kg \cdot a$$
$$a = \frac{113}{40}\,m/s^2 = \boxed{2{,}83\,m/s^2}$$

$$T = 15\,kg \cdot 2{,}83\,m/s^2 = \boxed{42{,}5\,N}$$

EJERCICIO 4 (1,5)



Para que se anulen debe estar aquí

$$\vec{F_2} + \vec{F_1} = \vec{0}$$
$$K \cdot \frac{q_2 q_3}{x^2} \cdot (-i) + K \cdot \frac{q_1 \cdot q_3}{(2+x)^2}\, i = 0$$
$$\cancel{K} \cdot \frac{q_2 \cancel{q_3}}{x^2}\, i = \cancel{K}\, \frac{q_1 \cancel{q_3}}{(2+x)^2}\, i$$
$$(2+x)^2 \cdot q_2 = q_1\, x^2$$

$$(2+x)^2 \cdot 3 \cdot \cancel{10^{-6}} = 8 \cdot \cancel{10^{-6}} x^2$$

$$(2+x)^2 = \frac{8}{3} x^2 = 2{,}7 x^2$$

Tomando raíces cuadradas

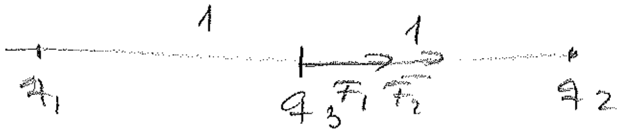
$$\sqrt{(2+x)^2} = \sqrt{2{,}7 x^2}$$

$$2+x = \sqrt{2{,}7}\, x = 1{,}64 x$$

$$2 = 1{,}64x - x = 0{,}64x$$

$$x = \frac{2}{0{,}64} m = \boxed{3{,}1 m}$$

b)



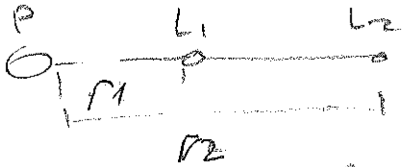
$$\vec{F} = \vec{F}_1 + \vec{F}_2 = K \frac{q_1 \cdot q_3}{d_1^2} i + K \frac{q_2 q_3}{d_2^2} i$$

$$= 9 \cdot 10^9 \cdot \frac{8 \cdot 10^{-6} \cdot 2 \cdot 10^{-6}}{1^2} i + 9 \cdot 10^9 \cdot \frac{3 \cdot 10^{-6} \cdot 2 \cdot 10^{-6}}{1^2} i \quad (N)$$

$$= (1{,}44 \cdot 10^{-1} i + 0{,}54 \cdot 10^{-1} i) N = 1{,}98 \cdot i\ 10^{-1} = \boxed{0{,}198\, i\ N}$$

c) Representado en las gráficas.

Ejercicio 6 (1,25)



Por la tercera ley de Kepler $\frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2}$

$r_2 = 2r_1$

$$\frac{r_1^3}{(2r_1)^3} = \left(\frac{T_1}{T_2}\right)^2$$

$$\frac{\cancel{r_1^3}}{2^3 \cdot \cancel{r_1^3}} = \left(\frac{T_1}{T_2}\right)^2 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \frac{1}{8} \Rightarrow \frac{T_1}{T_2} = \frac{1}{\sqrt{8}}$$

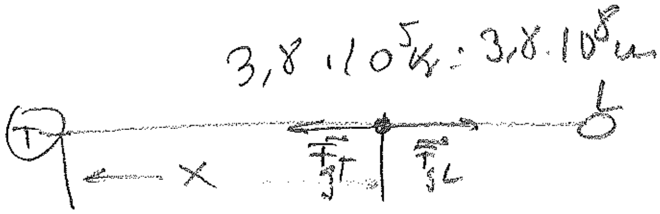
$$T_2 = \sqrt{8}\, T_1 = 2\sqrt{2}\, T_1$$

$$\frac{V_1}{V_2} = \frac{\frac{2\pi r_1}{T_1}}{\frac{2\pi r_2}{T_2}} = \frac{\frac{\cancel{2\pi} r_1}{T_1}}{\frac{\cancel{2\pi} r_2}{T_2}} = \frac{r_1 \cdot T_2}{r_2 \cdot T_1} = \frac{\cancel{r_1} \cdot 2\sqrt{2}\cancel{T_1}}{2\cancel{r_1} \cdot \cancel{T_1}} = \sqrt{2}$$

$$\boxed{V_1 = \sqrt{2}\, V_2}$$

EJERCICIO 7   1,75

a)



Hay un punto x en que las fuerzas se anulan

$$|\vec{F}_T| = |\vec{F}_L|$$

$$\frac{\cancel{G}\cdot M_T \cdot \cancel{m}}{x^2} = \frac{G \cdot M_L \cdot \cancel{m}}{(3{,}8\cdot 10^8 - x)^2}$$

$$\frac{81\, \cancel{M_L}}{x^2} = \frac{\cancel{M_L}}{(3{,}8\cdot 10^8 - x)^2}$$

$$81\,(3{,}8\cdot 10^8 - x)^2 = x^2$$

Tomando raíces cuadradas

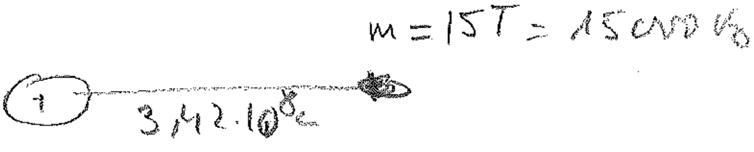
$$9\cdot(3{,}8\cdot 10^8 - x) = x$$

$$3{,}42\cdot 10^9 - 9x = x$$

$$3{,}42\cdot 10^9 = x + 9x = 10x$$

$$x = \frac{3{,}42\cdot 10^9}{10} = \boxed{3{,}42\cdot 10^8\ m}$$

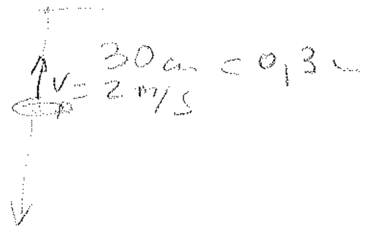
b)



$$F_g = \frac{G \cdot M_T \cdot m}{d^2} = \frac{6{,}67\cdot 10^{-11}\cdot 5{,}972\cdot 10^{24}\cdot 15000}{(3{,}4\cdot 10^8)^2}\ N$$

$$= \boxed{51{,}9\ N}$$

Ejercicio 4 (1,5)



a) $V_{max} = A\omega$

$$\omega = \frac{V_{max}}{A} = \frac{2\ m/s}{0,3\ m} = 6,7\ rad/s$$

b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{6,7}\ s = 0,94\ s$

c)

$$y(t) = A\cdot sen(\omega t + \varphi_0)$$

Para $t=0$ $x=0$ y $V = V_{max}$ hacia arriba

$$y(0) = 0 = A\cdot sen(\omega\cdot 0 + \varphi_0) = A\cdot sen(\varphi_0) = 0$$

$$\Rightarrow \varphi_0 = 0 \text{ o } \varphi_0 = \pi$$

$$V_{max} = A\omega\cdot\cos(\omega 0 + \varphi_0) = A\cdot\omega\cdot\cos(\varphi_0)$$

Como V_{max} es positiva $\Rightarrow \varphi_0 = 0$

$$\boxed{y(t) = 0,3\ sen(6,7\cdot t + 0)\ m}$$

d) $a_{max} = A\omega^2 = 0,3\cdot 6,7^2\ m/s^2 = \boxed{13,5\ m/s^2}$